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A comparison of two-dimensional strain analysis methods using elliptical grains

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Abstract—Two groups of methods of measuring the strain ellipse from elliptical grains are applied to deformed slates and sandstones, and the results are compared against one another. Group I includes the slope method and the method of means that measures the arithmetic, geometric, and harmonic means of grain axial ratios. Group II includes more sophisticated methods, and includes R_f/ϕ, the method of Shimamoto & Ikeda, the Robin method and a numerical version of the polar graph. This study shows that among group I, the harmonic mean method best approximates the strain ratio (R_s) at moderate strain. Among group II, there is a good linear correlation among R_s calculated by the Robin method, Shimamoto & Ikeda, R_f/ϕ, and to a lesser extent, the numerical polar graph. A graph incorporating the results of this study permits simple calculation of R_s from the harmonic mean of grain axial ratios for deformed rocks with similar lithologic and deformational characteristics to the samples used.

INTRODUCTION

AMONG strain markers, the shape and orientation of elliptical grains in plane sections of deformed rocks such as slate, sandstone, and conglomerate have been used extensively to measure the shape and orientation of the strain ellipse (e.g. Ramsay 1967, Dunnet 1969, Elliott 1970, Shimamoto & Ikeda 1976, Robin 1977, Lisle 1979). The final axial ratio and orientation of the grains depend on several factors: (1) the initial grain axial ratio (R_i), (2) the initial grain orientation, (3) the ductility contrast between grains and between grains and matrix, (4) orientation and axial ratio of the deviatoric strain, (5) volumetric strain and (6) incremental rotations (Ramsay 1967, Dunnet 1969, Gay 1968a,b, 1969, Elliott 1970).

In this paper, I present the results of several of the most common two-dimensional methods applied to deformed slates and sandstones of Upper Paleozoic rocks of the Golconda allochthon in the southern Toiyabe Range, Nevada, U.S.A. (Speed 1977, Babaie 1984). An attempt is made to compare the results and find a relationship among these using the various methods.

METHODS OF TWO-DIMENSIONAL STRAIN MEASUREMENT

The Golconda allochthon, in the study area, comprises several fault-bounded packets of slate, chert, turbiditic sandstone and basaltic rocks (Babaie 1984). The rocks studied were from the structurally lowest packets that include foliated slate and sandstone. These rocks were deformed by a first-phase isoclinal folding of bedding, during which slaty cleavage in pelitic rocks and spaced cleavage in wacke sandstones were developed. The cleavages are defined by planar orientation of micaceous minerals and flattened quartz and chert grains. The dimensions of the long and short axes of between 70 and 108 grains were measured in each section using an ocular micrometer and a petrographic stage. The trace of cleavage or an arbitrary line was used as a reference on sections normal and parallel to cleavage, respectively.

Only a general description of the methods used in this study is given here and the reader may consult the references for further details. Each method employs the ratio of the long and short axes of deformed grains (R_f) and the angle between the long axis of the grain and the reference lines (ϕ). In order to minimize the effect of the ductility contrast on strain determination, only quartz grains were measured in slates and chert grains in sandstones. The methods are categorized into two groups. Group I includes: (1) the slope method (Cloos 1947, 1971, Ramsay 1967), in which the slope of a best fit line passing through the origin of a plot of long versus short axis is taken as an estimate of the axial ratio of the strain ellipse (R_s) and (2) the method of means, in which the arithmetic, geometric, and harmonic means of the grain axial ratios (R_f) are used to approximate R_s (e.g. Cloos 1947, 1971, Ramsay 1967, Lisle 1977, Hossack 1968).

Group II includes: (1) the method of Shimamoto & Ikeda (1976) in which the R_f and ϕ data are used numerically to find the average final shape matrix and its eigenvalues to represent the principal axes of the strain ellipse; (2) the numerical version of the polar graph method of Elliott (1970), in which the strain axial ratio and orientation are statistically rather than visually determined by averaging the x-coordinates on the polar graph (Tobisch *et al.* 1977); (3) the method of Robin (1977), in which R_f and ϕ are used numerically to determine R_s and (4) the numerical version of the R_f/ϕ method (Ramsay 1967, Dunnet 1969, Dunnet & Siddans 1971, Peach & Lisle 1979), in which R_s is determined from the distribution of the grains on the R_f/ϕ plot by

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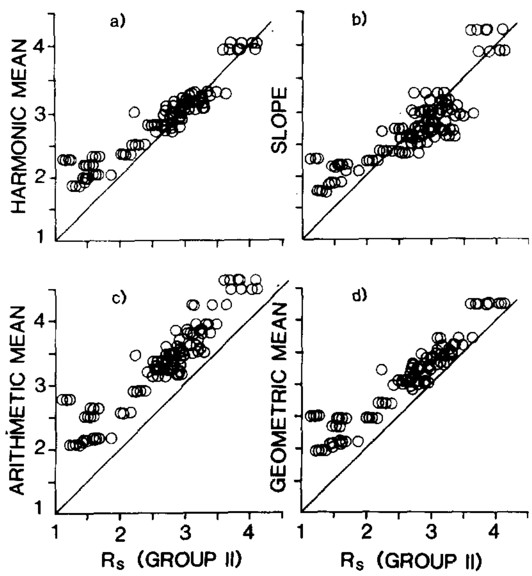

Fig. 1. Relationship between R_s of all group II methods and R_s estimated by the method of (a) harmonic mean, (b) slope, (c) arithmetic mean and (d) geometric mean of group I. Each point denotes results of a single method for each thin section.

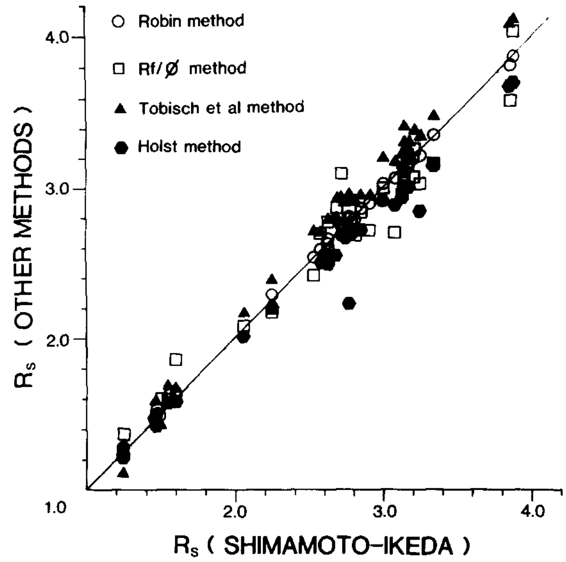

Fig. 2. Relationships between R_s calculated by the Shimamoto & Ikeda method and the Robin, R_f/ϕ, Tobisch *et al.* and Holst methods in group II.

comparing it with theoretical curves using the Chi-square test.

Tobisch *et al.* (1977) adjusted R_s that was calculated using the numerical version of the polar graph (Elliott 1970) by examining equivalent undeformed rocks in their study area. Where such undeformed rocks do not exist to provide a standard, the method leads to an overestimation of R_s. To compensate for this, Holst (1982, fig. 7) provided an empirical factor, F, that ranges between 0.87 and 1.00. The uncorrected R_s determined by the Tobisch *et al.* (1977) method may be multiplied by an appropriate value of F to provide a better measure of the two-dimensional strain.

The methods of the first group, not taking into account the initial grain axial ratio and orientation, provide an imprecise estimate of R_s unless grains are originally circular and deformed homogeneously (Ramsay 1967, Lisle 1977). It has been suggested that if grains with R_i between 1 and 4, and with no preferred orientation, are intensely but homogeneously strained, a good estimate of strain ratio can be obtained from the harmonic mean of their axial ratios (Lisle 1977, 1979, Borradaile 1984). The methods of the second group, in contrast, partly account for the factors that control the final shape of the grains, as mentioned above, and therefore provide a better estimate of the shape and orientation of the strain ellipse. In this study, R_s values calculated by group II methods are assumed to be correct and are compared against values calculated by each of the group I methods. The general assumption for all the methods is that grains and their matrix were deformed homogeneously.

RESULTS AND IMPLICATIONS

In general, the short and long axes of quartz grains in slate specimens lie between 0.003 and 0.112 mm, and 0.017 and 2.6 mm, respectively. The short and long axes of chert grains in sandstones range between 0.028 and 1.2 mm, and 0.07 and 5.88 mm, respectively. There is a good linear correlation between the long axes and short axes of all the grains, indicating that grains had nearly constant initial axial ratio and orientation prior to deformation. The correlation is better defined for smaller grains. This suggests a wider range of R_i values for larger grains and/or larger strain for the smaller grains. The grain-supported sandstones record a larger strain than the matrix supported slates, suggesting that strain was partly taken up by the matrix in the slates. The average R_i in the majority of the samples lay between 1.7 and 1.9 as measured by the polar graph and R_f/ϕ methods. However, individual grains with R_i less than 1.5 are common.

Strain ratios estimated by the arithmetic and geometric means are greater than those determined by methods of group II for all sections with low R_s values (Figs. 1c & d). However, in sections with large R_s, the difference between the results of the two groups of methods are close but not identical. The harmonic mean, in contrast, approaches the value of the strain ratio determined by all group II methods at moderate strain ($R_s > 2.5$) (Fig. 1a). Although the results of the slope and the group II methods overlap in sections with moderate R_s, the slope method yields more scatter than the harmonic mean method (Fig. 1b).

The difference between the results of group I and II methods is larger for grains with higher initial axial ratio, and is maximum for grains that had their axes within 5° of the major axis of the strain ellipse before deformation. This conclusion is based on results produced by the polar graph and R_f/ϕ methods and suggests that sedimentary fabrics such as an initial grain axial ratio and orientation, have significant effects on the results of group I methods.

Paterson (1983) and Wheeler (1984) concluded that the method of Shimamoto & Ikeda (1976) is the best among non-graphic methods using elliptical grains for strain analysis. Figure 2 shows the relationship between

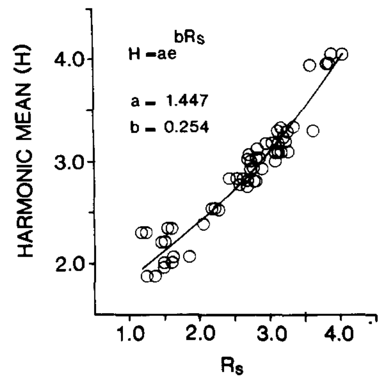

Fig. 3. Relationship between the harmonic mean and R_s calculated by the Shimamoto & Ikeda, Robin and R_f/ϕ methods. The solid line and its equation denote the best fit found by least squares.

R_s calculated by the method of Shimamoto & Ikeda (1976) and R_s found by other methods of group II. There is a good linear correlation between results of the Shimamoto & Ikeda (1976) and Robin (1977) methods. R_s determined by the numerical polar graph method (Tobisch *et al.* 1977) and adjusted using the empirical factor of Holst (1982) do not correlate as well with R_s of the Shimamoto & Ikeda method. At moderate to high strain, the Tobisch *et al.* (1977) method yields higher R_s values than those of the Shimamoto & Ikeda (1976) method, due to the overestimation involved. The R_f/ϕ method yields results that correlate reasonably well with those of the Shimamoto & Ikeda (1976) method.

The harmonic mean is calculated more readily than R_s of the group II methods and approximates the strain ratio better than any other group I method. A graph incorporating the results of this study has been constructed using the harmonic mean values and R_s determined by the methods of Shimamoto & Ikeda, Robin, and R_f/ϕ (Fig. 3). The best fit curve through data points on this plot has been found by the least squares method and is shown with its equation in Fig. 3. This graph may be used to estimate R_s from the harmonic mean of grain axial ratios in deformed rocks that have similar lithologic and deformational characteristics to the samples used in this study. The results of this study are consistent with those of Lisle (1977, 1979) and Borradaile (1984) who suggested that application of the harmonic mean method provides a good estimate of the strain ratio in cases where R_s is larger than 2.5 and where immediate application of other methods is not feasible.

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